The Energy Dependence of the Imaginary Nuclear Potential

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Recently the inverse mean field method (Imefim) has been shown to lead to a specific energy dependence of the real central nuclear part, V, of the optical model potential for nucleon-nucleus scattering. Proceeding from V towards the imaginary volume term of the potential, W_v, application of conservation laws yields a specific energy dependence for W_v. It compares favourably with heuristic potentials.

1. Motivation

Recently inverse methods have been applied to the nuclear bound-state problem [1, 2] leading – in liaison with concepts borrowed from nonlinear dynamics – even to a time dependent approach [3, 4] contrasting the traditional way of thinking. It has been termed the inverse mean field method (Imefim). The particular point, which we would like to discuss, relates to the optical model potential for elastic nucleon-nucleus scattering:

$$V_{\mathrm{om}} = V_{\mathrm{Cl}} + V + i\,W_{\mathrm{v}} + i\,W_{\mathrm{s}} + V_{\mathrm{so}}\,. \tag{1}$$

The different terms correspond to Coulomb interaction, real central (nuclear) part of the potential, imaginary volume and surface terms and spin-orbit contributions, respectively. Here we are only interested in V and W_{v}.

The local potential, V, is well known to be a function of the kinetic energy, E_{p}, of the incident projectile, and it is usually [2, 5] written in the form

$$V(r, E_{\mathrm{p}}) \cong V(r, 0)\, f(E_{\mathrm{p}}) = U(r)\, f(E_{\mathrm{p}})\,, \tag{2}$$

where r is the radial variable and $U(r)$ represents the shell-model potential of the target nucleus. In [6] it has been indicated that Imefim [1–4] predicts the energy dependence, $f(E_{\mathrm{p}})$, to obey

$$f(E_{\mathrm{p}}) = (1 + e\,E_{\mathrm{p}})^{-3} \quad \text{with} \quad e \cong 0.0027\ 1/\mathrm{MeV}. \tag{3}$$

The numerical value of e is (for the time being in the absence of a microscopic derivation) determined heuristically.

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In this short note we would like to show that (3) together with the application of conservation laws (that are well known to hold within the context of the nonlinear field equation of Imefim) lead to a specific energy dependence of the imaginary volume term, $W_{\mathrm{v}}(0, E_{\mathrm{p}})$. In other words, $W_{\mathrm{v}}(0, E_{\mathrm{p}})$ is to be derived from $V(r, E_{\mathrm{p}})$.

2. Derivation of $W_{\mathrm{v}}(0, E_{\mathrm{p}})$

In the derivation of $V(E_{\mathrm{p}})$ presented in [6] an average potential

$$U(x, E_{\mathrm{p}}) \equiv \bar{U}(x, E_{\mathrm{p}}) = -\,f(E_{\mathrm{p}})\, U_0\, \mathrm{sech}^2(x\sqrt{U_0\, m}/\hbar)\,; \quad U_0 = \mathrm{const};$$

has been used to facilitate the discussion. (Actually, the three dimensional counterpart of (4) should be used. However, as discussed in the note added in proof, the final result (8) as derived below does in now way depend on the specif spatial dependence of U.) Application of the exact expression for the shell-model potential [1–4] would just yield different absolute numbers without influencing the energy dependence, which is the only quantity we are really interested in. Hence, we use here also the simple expression (4).

It may be of interest to recall that sech^2 approaches for small arguments a Gaussian and for large ones a Saxon-Woods function. In view of the usage of these formfactors within nuclear physics it appears indeed sensible to employ (4) as a schematic representation for the nuclear potential. A further point of note is that (4) coincides essentially with

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the "one-level" approximation to Imefim which has been demonstrated [1] to yield useful and surprisingly accurate information related to absolute and relative nuclear (charge rms) radii.

Equation (4) implies [1, 6] that shell-effects are ignored. Therefore it is understood that we have no chances to extract from (3), (4) any information related to the imaginary surface term, W_s, which is usually associated with the suppressed shell-effects. But we may expect to learn something about the energy dependence of the volume absorption term, W_v.

Historically, the initially real scattering potential, V_{om}, has been supplemented in a phenomenological way by the imaginary volume term. It is to account for and to represent that part of the incident flux which is removed from the elastic channel into the inelastic ones. In other words, that part of the flux which we are *not* interested in, is catered for by the imaginary potential, W_v.

Let us now turn to Imefim and recall that colliding nuclei have been described in a TDHF-like fashion, see [4] and references. A distinct feature of Imefim is that in the identity

$$\int_{-\infty}^{\infty} V(x, E_p = 0)\, dx = \int_{-\infty}^{\infty} [V(x, E_p) + \{V(x,0) - V(x, E_p)\}]\, dx \quad (5)$$

the expression in the braces has to be identified with the flux removed from the elastic channel. Such a picture is fully in line with the spirit of the optical model (see also the formal and sophisticated considerations of [7]). Since these contributions are within the optical model accounted for via W_v, it appears most natural and consequent to identify them (i.e., the terms of (5) which are within the braces) with the spatial integral over W_v:

$$\int_{-\infty}^{\infty} W_v(x, E_p)\, dx = [1 - f(E_p)] \cdot \int_{-\infty}^{\infty} V(x,0)\, dx = [1 - f(E_p)]\, \hbar \sqrt{U_0/m2}\,. \quad (6)$$

To arrive at (6), use has been made of (2), (4) and (5). By assigning to W_v the spatial dependence indicated by (4), i.e.

$$W_v(x, E_p) = -g(E_p)\, W_0 \operatorname{sech}^2(x\sqrt{W_0 m}/\hbar)\,; \quad W_0 = \text{const}; \quad (7)$$

one obtains immediately

$$g(E_p) = 1 - f(E_p) = 1 - (1 + eE_p)^{-3}; \quad e \cong 0.0027\ 1/\text{MeV} \quad (8)$$

for the energy dependence of W_v. Under the assumption that there are no sources giving rise to additional flux, $W_0 = U_0$ is seen to hold. This is compatibel with the notion that $V(x, E_p)$ is a conserved quantity (see below). (In previous discussions [3] a slightly different formula has been shown, i.e., $g'(E_p) = [1 - (1 + eE_p)^{-3/2}]^2$; it arises if one assumes that the W_0 in the argument of (7) has the same energy dependence as the amplitude of sech^2.) From (2) and (5), (6) it is obvious that different geometries for the potential W_v do not influence the energy dependence (if (2) is justified as usually assumed) – but for a constant renormalization coefficient.

Before going over to the discussion of (8); let us present a rather unconventional view at such scattering experiments. It is to give rise to the same result, (8). Usually nucleon-nucleus scattering experiments are analyzed in terms of the *time independent* stationary Schrödinger equation. However, we may as well adopt a slightly different point of view:

Say, we have a single accelerator at our disposition. Its source provides a well specified constant flux. To study projectile energy dependent effects, we re-do the same experiment at different projectile energies – which has in the depicted situation necessarily to be done at different times. In the sense that E_p sets the initial conditions for the different experiments, it may thus be considered to be a "function" of time, $E_p(t)$, see also [4]. A conservation law that is well known to hold within the context of the nonlinear field equation of Imefim (see [3, 4] and references) states that $V(x, E_p(t))$ is a conserved quantity. In view of (2) to (4) this insinuates that the volume integral over the shell-model potential, (5), is a constant in respect to changes in the variables E_p and/or t. – Compared to the traditional way of thinking such an interpretation of scattering experiments appears not just unusual but even artificial. Yet, it leads to the same notion as discussed above: since (5) is a conserved quantity and since $\int_{-\infty}^{\infty} V(x, E_p)\, dx$ is to be associated with the elastic channel, we have to interprete the remaining contributions (in braces, see (5)) in terms of flux

transferred into other (i.e., inelastic) channels, i.e., in terms of W_v. The consequent development of these arguments leads then again to (8).

3. Discussion

In [6] the energy dependence of $V(E_p)$ due to Imefim, (3), has been compared to heuristic data and to results based on dispersion relations (both taken from [8]). Dispersion relations and Imefim lead to curves for $V(E_p)$ of the same quality as far as comparison to the heuristic data is concerned.

In [8] dispersion relations have been demonstrated to yield also $W_v(E_p)$. However, the empirical data are only reproduced for lower and intermediate energies, see Figure 1. For larger projectile energies the dash-dotted curve due to dispersion relations fail completely.

The other two curves shown in Fig. 1 are based on the present approach, (8), with the same parameters as used previously [6] in the discussion of $V(E_p)$. There is no arguing against the fact that (8) predicts correctly the qualitative features of $W_v(E_p)$. But the same statement does not hold for the quantitative behaviour of these curves: At lower energies up to about 70 MeV, Imefim is even closer to the phenomenological data than dispersion relations. But in the range from 70 MeV to 500 MeV the opposite is true. However, at very large energies dispersion relations fail completely while (8) is in a very nice agreement with the data.

Pondering about the quality of these curves and about the physics and mathematics associated with dispersion relations and Imefim, the latter appears to be superior since it facilitates also a more comprehensive view at nuclear physics [1–4, 6]. In addition to that a glance at Fig. 1 gives rise to the hope to bring about some quantitative changes in the description of Imefim (in contrast to the qualitative and quantitative changes that would be required in the case of dispersion relations). Following this train of thought, it has to be recalled that Imefim is based on the *nonrelativistic* Schrödinger equation; hence, the need for relativistic corrections at higher energies is to be anticipated. To clarify the situation (in particular at intermediate energies) and to remove possible ambiguities it will be necessary to consider some more recent heuristic potential sets, which have to be treated with care to ensure that they are really compatible with each other. It can not be excluded that this is also to lead to some corrections of the data discussed in Figure 1. A further point is that it is possibly more reasonable to include all the potential parameters (or at least the respective volume integrals) in the discussion. Such work is still in progress.

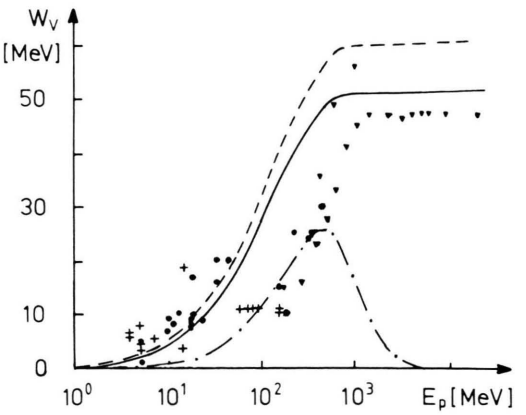

Fig. 1. The depth $W_v(0, E_p)$ of the imaginary volume absorption potential is plotted as a function of the projectile energy, E_p. The heuristic data (points, triangles, etc.) and the dash-dotted curve based on dispersion relations stem from [8] which should be consulted for further details on them. Full and dashed curves correspond to (8) with the same parameters as used previously [6] (i.e., $e = 0.0027$ 1/MeV and $U(0, 0) = 52.5$ MeV and 61.5 MeV).

4. Summary

Starting with the energy dependence of the real central nuclear part of the optical model potential as given previously [6], a conservation law and/or other considerations [4] lead to the notion to associate the energy dependent reduction in $\int V(x, E_p)\, dx$ with flux removed from the elastic channel. This interpretation leads to a unique energy dependence, (8), for the imaginary volume term, $W_v(x, E_p)$. For the limited data considered here, $W_v(0, E_p)$ is in very nice qualitative (and in reasonable quantitative) agreement with phenomenological potentials – in contrast to the results of dispersion relations.

In view of the problems and tedious calculations associated with conventional approaches, it is note worthy that the interpretation of $V(x, 0)$ as a conserved quantity yields almost immediately such a nice and simple result. It is in particular encourag-

ing that no further assumptions are required to extract from $V(x, E_p)$ a unique energy dependence for the imaginary volume term, (8). Yet, to substantiate the proposed interpretation free of doubt, to make it generally accepted and to improve it quantitatively a lot more work remains to be done*.

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* *Note added in proof:*

The final result of this paper, i.e., the $g(E_p)$ of (8), does not depend on a particular choice for the spatial dependences of V, U, and W_v – as long as they are all the same ones. In particular, it is for (8) irrelevant whether one dimensional (variable x and volume element dx) or three dimensional (radial variable r and its volume element $r^2\,dr$) cases are considered. To stress this arbitrariness the spatial variable(s) is (are) now denoted by $\hat{x}$ and the respective volume element by $d\hat{x}$. Using in line with (1) the relation $V(\hat{x}, U_0; E_p) = U(\hat{x}, U_0) \cdot f(E_p)$ in (5) and (6) one arrives in analogy to (6) immediately at

$$\int W_v(\hat{x}, E_p)\,d\hat{x} = [1 - f(E_p)] \int V(\hat{x}, 0)\,d\hat{x} = [1 - f(E_p)]\, h(U_0)\,, \qquad (6')$$

where the specifics of the function $h(U_0)$ are pre-determined by the geometry of $U(\hat{x}, U_0)$. Employing instead of (7) the general expression $W_v(\hat{x}, W_0; E_p) = W_v(\hat{x}, W_0; 0)\, g(E_p)$ one is again lead to (8). However, this simple derivation depends crucially on the validity of the factorization of $V(\hat{x}, E_p)$ and $W_v(\hat{x}, E_p)$ into functions of $\hat{x}$ and E_p. According to the present wisdom this is a reasonable assumption, see also [2, 5] and references. – Different geometries for U and W_v, say, do not invalidate the derivation of (8), they give only rise to renormalizations of the energy dependence $g(E_p)$, e.g., $g'(E_p) = c\, g(E_p)$ where c is the respective re-normalization constant.

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